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Fibonacci Numbers and Their Place in Life

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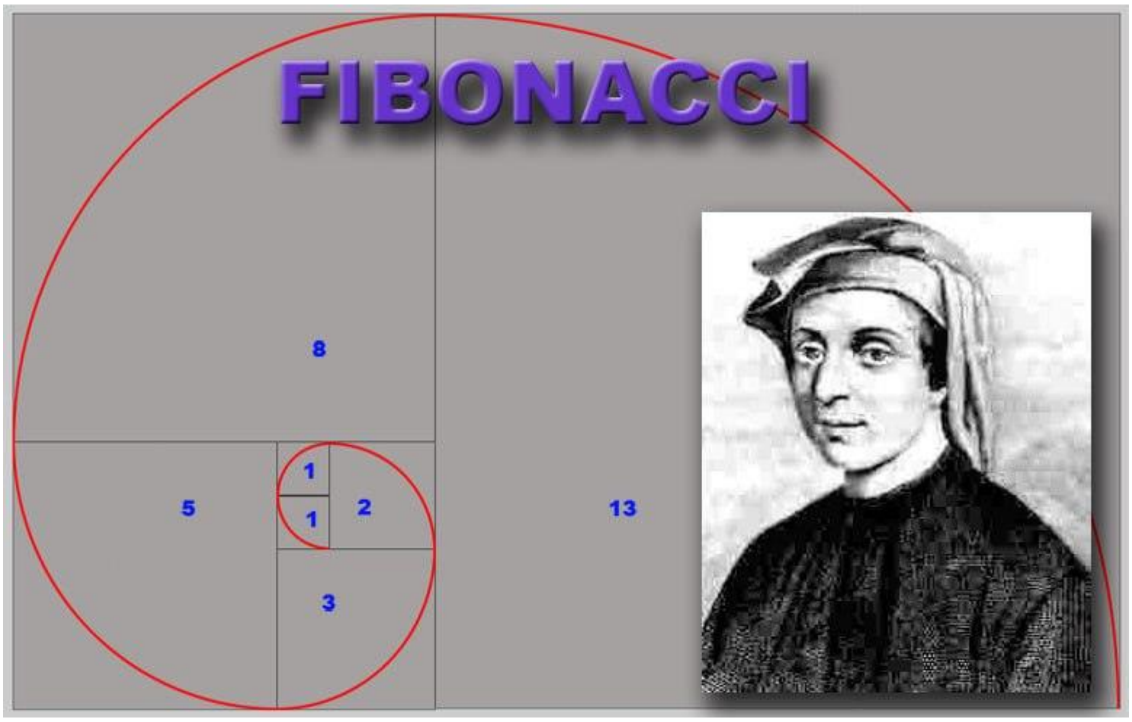
Abstract: *this article details the sequence of Fibonacci numbers, its historical origins, mathematical properties, and application in practice.*

Keywords: *Fibonacci numbers, Golden Ratio, Proportion, builder Matrix, application of Fibonacci numbers in nature.*

Introduction.

Although the Fibonacci numbers are named after Leonardo of Pisa (also known as Fibonacci), one of the greatest European mathematicians of the Middle Ages, who was born around the year 1175, these numbers actually originated long before his time. The sequence of numbers—1, 1, 2, 3, 5, 8, 13, 21, …—formed based on a specific pattern, is called the Fibonacci sequence. These numbers were first used in ancient India. According to Singh, the Indian mathematician Acharya Pingala was the first to have knowledge of Fibonacci numbers. It is believed that he lived around 400 BCE. The Indian mathematician Acharya Virahanka, who lived around 600–800 CE, is considered the first to have presented the Fibonacci sequence in written form.



Definition: The Fibonacci numbers are a set of numbers in which each number, is expressed as the sum of the two preceding terms.

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Fibonacci sequence continues as follows: 1, 1, 2, 3, 5, 8, 13, …

That is, in this case, F(n) is defined as F(n−1) + F(n−2). $n \ge 2$ In this case, $F(0)=0$, $F(1)=1$ is defined as

N	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**…**
F(n)	0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	…

The Fibonacci sequence can also be expressed using a matrix. To do this, we construct the following generating matrix:

$$M = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$$

Using this matrix, we can find the Fibonacci numbers using the following formula: $\begin{pmatrix} F_n \\ F_{n-1} \end{pmatrix} = M^{(n-1)} \cdot \begin{pmatrix} F_1 \\ F_0 \end{pmatrix}$

Now let's consider the connection between Fibonacci numbers and the golden ratio (section). The golden ratio, also known in mathematics as the 'golden section,' is an important ratio widely used in the field of mathematics. This ratio is approximately equal to 1.61803 and is denoted by the Greek letter φ (phi). The value of the golden ratio is given in mathematical form as follows: $\varphi = \dfrac{1+\sqrt{5}}{2}$

First, we write out the Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, … Then, we examine the ratio for each Fibonacci number. $\dfrac{F_{n+1}}{F_n}$

Nomer	F_{n+1}/F_n	F_{n+1}/F_n **numerical value of the expression**
1	$1 \div 1$	1
2	$2 \div 1$	2
3	$3 \div 2$	1.5
4	$5 \div 3$	1.66666666…
5	$8 \div 5$	1.6
6	$13 \div 8$	1.625
7	$21 \div 13$	1.61538562…
8	$34 \div 21$	1.61904762…
9	$55 \div 34$	1.61764706…
10	$89 \div 55$	1.61818182…
11	$144 \div 89$	1.61797753…
12	$233 \div 144$	1.6180556…

As we can see, in the Fibonacci sequence, the ratio of two consecutive numbers gradually approaches the golden ratio. The mathematical connection between Fibonacci numbers and the golden ratio: $\lim\limits_{n\to\infty} \dfrac{F_{n+1}}{F_n} = \varphi$ here, φ - represents the golden ratio. Let's list some properties of the Fibonacci numbers:

1°. $F_1 + F_2 + F_3 + ... + F_n = F_{n+2} - 1$

Proof: $F_3 = F_1 + F_2 \;\Rightarrow\; F_1 = F_3 - F_2$

$F_4 = F_3 + F_2 \Rightarrow F_2 = F_4 - F_3$

……………………………………………………………….

$F_{n+2} = F_n + F_{n+1} \Rightarrow F_n = F_{n+2} - F_{n+1}$

$F_1 + F_2 + F_3 + ... + F_n = F_3 - F_2 + F_4 - F_3 + ... + F_{n+1} - F_n + F_{n+2} - F_{n+1} = F_{n+2} - 1$

$F_1 + F_2 + F_3 + ... + F_n = F_{n+2} - 1$

2°. $F_1 + F_3 + F_5 + F_7 + ... + F_{2n-1} = F_{2n}$

Proof: $F_1 = F_2 = 1$

$F_1 + F_3 + F_5 + F_7 + ... + F_{2n-1} = F_2 + (F_4 - F_2) + (F_6 - F_4) + ... + (F_{2n} - F_{2n-2}) = F_{2n}$

$F_1 + F_3 + F_5 + F_7 + ... + F_{2n-1} = F_{2n}$

3°. $F_2 + F_4 + F_6 + ... + F_{2n} = F_{2n+1} - 1$

Proof: $F_2 + F_4 + F_6 + ... + F_{2n} = F_1 + F_2 + F_3 + ... + F_{2n} - \left(F_1 + F_3 + F_5 + F_7 + ... + F_{2n-1}\right) =$
$F_{2n+2} - 1 - F_{2n} = F_{2n+1} - 1$

4°. $F_1 - F_2 + F_3 - F_4 + ... + (-1)^{n+1} F_n = (-1)^{n+1} F_{n-1} + 1$

5°. $F_1^2 + F_2^2 + ... + F_n^2 = F_n F_{n+1}$

$F_1 = F_2$

$F_2 = F_3 - F_1$

$F_3 = F_4 - F_2$

…………………………

$F_n = F_{n+1} - F_{n-1}$

$F_1^2 + F_2^2 + ... + F_n^2 = F_1 F_2 + F_2(F_3 - F_1) + F_3(F_4 - F_2) + ... + F_n(F_{n+1} - F_{n-1}) = F_n F_{n+1}$

6°. $F_1 F_2 + F_2 F_3 + F_3 F_4 + ... + F_{2n} F_{2n+1} = F^2{}_{2n+1} - 1$

7°. $F_1 F_2 + F_2 F_3 + F_3 F_4 + ... + F_{2n-1} F_{2n} = F_{2n}^2$

The Fibonacci sequence originates from the famous 'rabbit problem' discussed in Fibonacci's book *Liber Abaci*. In the book, the problem is presented as follows: 'If each pair of fertile rabbits produces a new pair beginning in the second month, starting with a single pair, how many pairs of rabbits will be produced each month?' In this problem, it is assumed that rabbits do not die and that each new pair of young rabbits becomes a mature (fertile) pair after one month. Another important note is that rabbits are not fertile until they are two months old.

The table below summarizes the number of adult pairs, young pairs, and the total number of rabbits each month. Each new pair of young rabbits is equal to the number of adult pairs from the previous month, since every adult pair produces one new pair per month. The numbers in the total column are known as Fibonacci numbers, and as the process continues, it generates the Fibonacci sequence.

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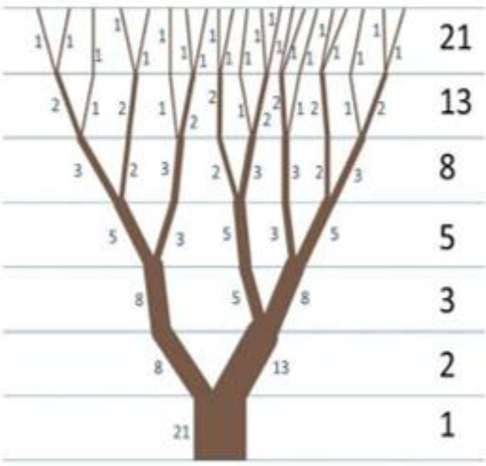
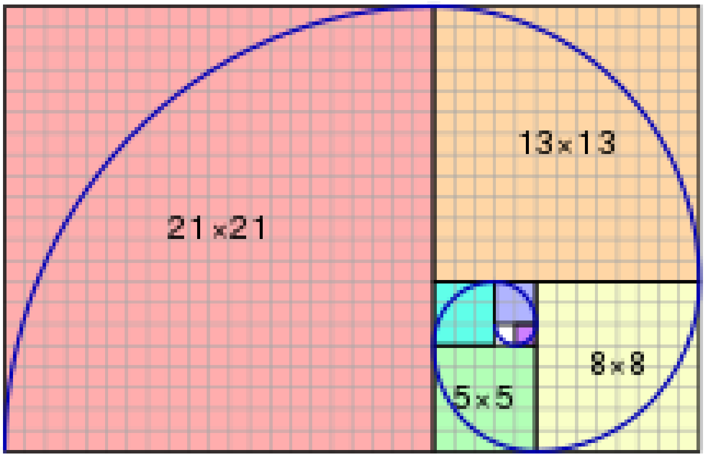
RABBITS, COLONY GROWTH			
Months	**Adult couples**	**Young couples**	**Total**
1	1	1	2
2	2	1	3
3	3	2	5
4	5	3	8
5	8	5	13
6	13	8	21
7	21	13	34
8	34	21	55
9	55	34	89
10	89	55	144
11	144	89	233
12	233	144	377

Fibonacci numbers frequently appear in nature, which further increases interest in them. These numbers are reflected in the growth patterns of plants. For example, rows of leaves, flowers, or fruits on plants are often arranged according to Fibonacci numbers. Typically, leaves are arranged in spirals, and their numbers are often 3, 5, 8, 13, and so on.



As an example of Fibonacci numbers, if circles with equal radii are drawn inside the following squares, a beautiful spiral is formed as shown below.



Conclusion

Based on the information presented in the article, Fibonacci numbers hold significant importance not only in mathematics but also in the realms of nature and art. In many ways, they help us gain a deeper understanding of the world around us. Fibonacci numbers form a sequence of natural numbers in which each number is defined as the sum of the two preceding ones. These numbers also appear in works of art and architecture, as they represent aesthetic harmony and beauty. Fibonacci numbers and their ratios incorporate the important mathematical concept known as the "golden ratio." Overall, Fibonacci numbers contribute not only to mathematical curiosity but also enhance our understanding of beauty in nature and art.

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